ASSIGNMENT SET – I

Mathematics: Semester-III

M.Sc (CBCS)

Department of Mathematics

Mugberia Gangadhar Mahavidyalaya

PAPER - MTM-301

Paper: Partial Differential Equations and Generalized Functions

1. Solve the following: $\left(D^2 + 5DD' + D'^2\right)z = 0$ where $D \equiv \frac{\partial}{\partial x}$ and $D' \equiv \frac{\partial}{\partial y}$.
2. Find the derivative of the Heaviside unit step function.
3. Find the solution of $z^2 = pqxy$.
4. Solve: $\left(x^2D^2 - 2xyDD' + y^2D'^2 - xD + 3yD'\right)u = 8\frac{y}{x}$. Symbols have their usual meaning.
5. Show that the Green function for the Laplace equation is symmetric.
6. Check the validity of the maximum principle for the harmonic function $\frac{1-x^2-y^2}{1-2x+x^2+y^2}$ in the disk $\overline{D} = \{(x, y): x^2 + y^2 \leq 1\}$.
7. Establish the Poisson's formula for the solution of a Dirichlet problem for the Laplace equation in a disk of radius a.
8. State and prove the strong maximum principle.
9. Establish the Laplace equation in polar coordinates.
10. Prove that $\delta(at) = \frac{1}{a}\delta(t)$, Symbols have their usual meaning.
11. Obtain the solution of the interior Dirichlet problem for the Poisson's equation in a sphere using the Green's function method. Hence derive the Poisson integral formula.
12. Define the Domain of dependence of the Cauchy problem for the wave equation.
13. Define characteristic curve and characteristic base curve of a first order quasi linear PDE.
14. Find the solution of $(D^2 - DD' - 2D'^2)z = (y-1)e^x$, Where $D = \frac{\partial}{\partial x}$ $and\ D' = \frac{\partial}{\partial y}$

15. Prove that the type of a linear second order PDE in two variables is invariant under a change of Co-ordinates.
16. State Basic existence theorem for Cauchy Problem.
17. Solve the following PDE: $\left(x^2D^2 - 4xyDD' + 4y^2D'^2 + 6yD'\right)z = x^3y^4$ Where $D = \frac{\partial}{\partial x}$ $and\ D' = \frac{\partial}{\partial y}$
18. Find the integral surface of the linear PDE $(x-y)p + (y-z-x)q = z$ which passes through the circle $x^2 + y^2 = 1, z = 1$.
19. Reduce the following equation to a canonical form and hence solve it
$$yu_{xx} + (x+y)u_{xy} + xu_{yy} = 0.$$
20. Find the general integral of the PDE $p^2y(1+x^2) = qx^2$.
21. Establish the d' Alembert's formula for solve the Cauchy problem for homogeneous wave equation.
22. Define Dirichlet boundary condition and Neumann boundary condition.
23. State and prove the strong maximum principle for elliptic pole.
24. Find the adjoint of the differential operator L(u) = $u_{xx} + u_{tt} - u_t$.
25. Find the PI of the PDE $(D^2 + DD' + D' - 1)z = \sin(x+2y)$.
26. What are the main difference between an ODE and PDE ?
27. Define Dirac delta function.
28. Eliminate the arbitrary function f and F from y = f(x- at) + F(x+at) .
29. Prove that every nonnegative harmonic function in the disk of radius a satisfies $\frac{a-r}{a+r}\, u(0,0) \leq u(r,\theta) \leq \frac{a+r}{a-r}\, u(0,0)$.
30. Solve the problem $u_{tt} - u_{xx} = 0 \quad 0 < x < \infty,\ 0 < t, \quad u(0,t) = \frac{t}{1+t}, \quad 0 \leq t$, $u(x,0) = u_t(x,0) = 0,\ 0 \leq t < \infty$
31. Let u(x, y) be an integral surface of the equation a(x, y)$u_x + b(x,y)u_y + u = 0$, where a(x, y) and b(x, y) are positive differentiable function in the entire plane. Define D = {(x, y): $|x| < 1$, $|y| < 1$}
    (i) Show that if u be positive on the boundary of D , then it is positive at every point in D.
    (ii) Suppose that u attains a local minimum (maximum) at a point $(x_0, y_0) \in D$. Find $u(x_0, y_0)$.
32. State the interior and exterior Dirichlet problem.
33. Find the solution of the following problem $u_x + x^2u_y = 0$ with $u(x,0) = e^x$.
34. Let $u(x,y)$ solve the Cauchy problem $\frac{\partial u}{\partial y} - x\frac{\partial u}{\partial x} + u - 1 = 0$ where $-\infty < x < \infty$, $y \geq 0$ and $u(x,0) = \sin x$. Then find the value of $u(0,1)$.
35. Consider the initial value problem $\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,\ u(0,y) = 4e^{-2y}$. Then find the value of $u(1,1)$.
36. Find the complete integral of the PDE $\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = xe^{x+y}$.

37. Let P(x, y) be a particular integral of the partial differential equation $\frac{\partial^z z}{\partial x^2} - \frac{\partial z}{\partial y} = 2y - x^2$. Then find the value of P(2, 3).
38. Let u be the unique solution of $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = 0, x \in R, t > 0,\ u(x,0) = f(x)$, $\frac{\partial u}{\partial t}(x,0) = 0,\ x \in R$. Where $f(x) = x(1-x), \forall x \in [0,1]$ and $f(x+1) = f(x) \forall x \in R$. Then find the value of $u(\frac{1}{2}, \frac{5}{4})$.
39. Let u(x, t) satisfy the initial boundary value problem $u_t = u_{xx}\,; x \in (0,1), t > 0$, $u(x,0) = \sin(\pi x)\,; x \in [0,1],\ u(0,t) = u(1,t0 = 0, t > 0$. Then find the value of $u\left(x, \frac{1}{\pi^2}\right),\ x \in (0,1)$.
40. Let u(x, t) be the solution of $\frac{\partial^2 u}{\partial t^2} - \frac{\partial^2 u}{\partial x^2} = xt, \quad -\infty < x < \infty, t > 0$, $u(x,0) = \frac{\partial u}{\partial t}(x,0) = 0, \quad -\infty < x < \infty$. Then find the value of u(2, 3).
41. Let u(x, t) satisfy the initial boundary value problem $\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} \quad 0 < x < \infty, t > 0$ $u(x,0) = \cos(\frac{\pi x}{2})\ , 0 \le x < \infty\ \frac{\partial u}{\partial t}(x,0) = 0, 0 \le x < \infty\ , \frac{\partial u}{\partial x}(0,t) = 0,\ t \ge 0$. Then find the value of $u(2,2)$ and $u(\frac{1}{2}, \frac{1}{2})$.

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